



Simple-II: A new numerical thermal model for predicting thermal performance of Stirling engines



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ABSTRACT

A new thermal model called Simple-II was presented based on modification of the original Simple analysis. First, the engine was modeled considering adiabatic expansion and compression spaces, in which effect of gas leakage from cylinder to buffer space and shuttle effect of displacer were implemented in the basic differential equations. Moreover, non-ideal thermal operation of the regenerator and the longitudinal heat conduction between heater and cooler through the regenerator wall were considered. Based on the magnitudes of pressure drops in heat exchangers, values of pressure in the expansion and compression spaces were corrected. Furthermore, based on the theory of finite speed thermodynamics (FST), the corresponding power loss due to the piston motion and also the mechanical friction were considered. Simple-II was employed for thermal simulation of a prototype Stirling engine. Finally, result of the new model was evaluated by comprehensive comparison of experimental results with those of the previous models. The output power and thermal efficiency were predicted with +20.7% and +7.1% errors, respectively. Also, the regenerator was demonstrated to be the main source of power and heat losses; nevertheless, other loss mechanisms have reasonable effects on output power and/or thermal efficiency of Stirling engines.

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1. Introduction

Stirling engines are one of the external heat transfer engines that can use various heat sources such as solar, bio-mass, convective fuel, nuclear and so on [1–3]. Stirling engines operate in a closed regenerative thermodynamic cycle with two working spaces (compression space and expansion space) and three heat exchangers including heater, cooler and regenerator. Compression takes place at a lower pressure level while heat is rejected to a cold heat sink through a heat exchanger called cooler. In addition, expansion process occurs at a higher pressure with heat addition from a heat source via another heat exchanger called heater. Working spaces of the engine are divided into expansion and compression cylinders with separate pistons and displacer. These pistons are used for displacing and distributing the working fluid between expansion and compression spaces. In modern Stirling engines, a regenerative heat exchanger called regenerator is

employed to recover heat from the working fluid passing between expansion and compression spaces.

Developing the Stirling engine for high thermal efficiency production of power has been the concern of many researchers around the world. Many studies have been performed for thermal modeling and prediction of thermal performance of Stirling engines, in which researchers have tried to present precise thermal models that enable them to predict thermal efficiency and output power of Stirling engines with the highest possible precision and low computational cost. These researches can be classified into empirical, analytical and numerical methods.

Several works can be cited here as examples for empirical models [4–8]. West [4] considered an output power of the Stirling engine and presented a new dimensionless number called Beal number. This number was used to predict output power of Stirling engines. Kongtragool and Wongwises [5] performed experimental analysis of Stirling engines and found that Beal number could not be used for predicting thermal performance of the engines working with high temperature ratios. Thus, they corrected Beal number formulation using a suitable correction factor. Prieto et al. [6–8] performed comprehensive experiments on Stirling engines and used similarity and dimensionless analysis method to create

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empirical correlations for predicting power and efficiency of Stirling engines.

Analytical models have been employed by a number of researchers as an option for predicting thermal performance of Stirling engines or Stirling cycles [9–16]. These analytical models are divided into two groups including methods for analyzing Stirling cycles [9–13] and methods for simulating Stirling engines [13–16]. The methods used for analyzing Stirling cycle are simple in application; but, they do not contain complicated engine parameters (such as dead volume, stork, heat exchanger data and so on). Therefore, they cannot be applied to the engine simulation in practical cases. These methods can be employed for thermodynamic analysis of Stirling cycles that approximately model a real engine. Finite time thermodynamics (FTT) and finite speed thermodynamics (FST) are two kinds of closed form analyses that are employed for Stirling cycles. Yaqi et al. applied FTT for isothermal analysis of solar Stirling engines and tried to optimize the working space temperature [9]. Later, Ahmadi et al. [10] used FTT model to optimize a solar Stirling cycle by simultaneously considering thermal efficiency and output power as objective functions. Petrescu et al. [11] and Costa et al. [12] performed finite speed analysis of the isothermal Stirling engine cycle and evaluated irreversibility of the Stirling engine cycle in terms of speed and friction of piston displacement. They considered adiabatic analysis as a basic ideal analysis and employed experimental relations for the irreversibility and pressure drops in the regenerator. Also, new equations were used to calculate the friction factor and pressure drops in heat exchangers. Terms of finite speed pressure drop and mechanical friction losses were applied as a different form in numerical model to calculate the power losses due to these terms as internal irreversibility. Ahmadi et al. [13] optimized a Stirling cycle for maximum efficiency and power and minimum pressure loss based on FST model.

The second category of analytical methods is the methods used for simulating real engines. These models consider engine parameters including volume, stork, crank rotation angle, phase difference between cylinders, drive mechanisms, parameters of heat exchangers and so on. Therefore, these models are more successful in simulating real engines, but are more complicated than cycle based models. Schmidt isothermal model [14] was the first analytical model introduced for determining the pressure distribution and performance prediction of Stirling engines. In Schmidt model, it was assumed that expansion and compression processes were isothermal. Toda et al. [15] modified Schmidt theory to include effect of drive mechanism type in the model and analyzed a gamma type Stirling engine based on the developed model. Their results implied that type of mechanical drives had a strong influence on thermal performance of Stirling engines. Martini [16] presented a new mathematical model based on Schmidt model, in which effects of pressure drop and heat loss were considered in predicting output power of Stirling engines.

Some numerical models based on thermodynamic principles have been considered for analysis and performance prediction of Stirling engines. Finkelstein [14], for the first time, developed a new model based on adiabatic expansion and compression processes. In this model, heat exchangers were assumed to work in an isothermal condition. Urieli and Berchowitz [17] tried to develop the adiabatic model of Stirling engines using numerical calculation. In their work, adiabatic model was numerically calculated and a computer code was developed for performance prediction of Stirling engines. Urieli and Berchowitz [17] developed a new model (so-called Simple analysis) to improve results of the adiabatic model. In their model, non-ideal heat transfers of exchanges were considered. Abbas et al. [18] developed a new analysis based on Simple model, in which effects of the non-ideal regeneration,

shuttle loss and heat conduction losses were considered. Effects of pumping work loss and regenerator heat loss were evaluated in a research conducted by Strauss and Dobson [19]. Granodos et al. [20] and Nepveu et al. [21] developed a new model based on quasi-steady flow approach for analyzing the SOLO 161 Stirling engine with alpha configuration. Parlak et al. [22] applied quasi-steady flow analysis for evaluating performance of a gamma-type Stirling engine. Tlili et al. [23] developed a second-order quasi-steady flow mathematical model for evaluating performance of medium-temperature differential Stirling engine. In their analysis, effects of shuttle loss, heat and pressure losses were considered. Effect of gas spring hysteresis losses in working spaces of Stirling engines was studied by Timoumi et al. [24]. In his analysis, effect of heat absorption and rejection between compression and expansion spaces with displacer movement (shuttle effect) was considered. Schulz et al. [25] presented a new general model of Stirling engines to predict power and efficiency. Ercan Ataer and Karabulut [26] used a mathematical model for the evaluation of a V-type Stirling-refrigerator without pressure drop analysis. Martaj et al. [27] analyzed a low-temperature differential Stirling engine at steady state operations. Robson et al. [28] presented a new mathematical analysis of a low-temperature differential Ringbom Stirling engine, in which thermal parameters in expansion, compression and regenerator space were considered.

Some researches have considered detailed design and evaluation of Stirling engines [29,30]. Cheng et al. [29] optimized the rhombic drive mechanism used in beta-type Stirling engines based on dimensionless analysis. Li et al. [30] considered effect of compact porous-sheets heat exchangers on performance of Stirling engine. In this work, heater, regenerator and cooler were formed by hundreds of porous metal sheets and identified for theoretical analysis to facilitate the future scale-up design.

In the present work, a new numerical thermal model called Simple-II model was presented, which was developed by modifying the original Simple analysis [17] previously presented for thermal simulation of Stirling engines by Urieli and Berchowitz. Similar to the original Simple analysis, working spaces were considered as adiabatic spaces. The system of differential equations of the Simple (Adiabatic) model was modified to include effect of heat absorption and rejection between expansion and compression spaces by displacer (known as shuttle conduction heat loss) and mass leakage between working and buffer spaces. Therefore, differential equations of mass and energy conservations of the original Simple analysis were modified to include terms of shuttle effect and mass leakage, respectively. Moreover, new forms of conservation equations were used to modify the differential equation related to pressure. The new corrected system of differential equations was numerically solved using fourth-order Runge-Kutta method [17]. Then, similar to the original Simple model, results of the numerical solution were corrected for non-ideal heat transfer and pressure drops in heat exchangers (heater, cooler and regenerator). In final modification of the method, the results obtained in the previous parts of calculation were modified for losses due to finite speed of the pistons (working pressure over two pistons was corrected), mechanical friction and longitudinal conduction loss between heater and cooler through the regenerator wall. For correcting working pressure on two pistons, method of finite speed thermodynamic was applied. The developed model was applied to the GPU-3 Stirling engine as a case study. This engine is a prototype 3.0 kW Stirling engine developed by General Motors. Thermal efficiency and output power predicted by the Simple-II thermal model were compared with those of the previous thermal models as well as experimental data reported by Lewis Research Center [31]. It was shown that Simple-II model could predict thermal performance of Stirling engines with less deviation from actual data

compared to the previous thermal models. Finally, effects of engine parameters on thermal performance of the engine were studied based on Simple-II thermal model. In summary, it can be said that novelties of this paper were in integrating various loss effects including shuttle effect, mass leakage effect, power losses due to finite speed of pistons and mechanical friction and longitudinal conduction heat loss through the regenerator wall into the original Simple analysis, which was previously presented for thermal simulation of Stirling engines.

2. Modeling

2.1. Basic adiabatic and simple analysis

Adiabatic expansion and compression spaces were the core of adiabatic and simple analyses. Adiabatic model was developed based on the following assumptions:

1. All processes are performed in a steady-state condition.
2. The engine is working at constant angular velocity.
3. The working fluid is ideal gas.
4. The engine is working with adiabatic compression and expansion spaces.
5. Instantaneous pressure of compression and expansion spaces are uniform.
6. Working fluid temperatures in the cooler and heater are constant.
7. Temperature of the working fluid in the regenerator is linearly changed.
8. The kinetic and potential energies of gas streams is negligible.
9. Heat is transferred to the working fluid only the in the heater and cooler.
10. Total mass of the working gas is constant.
11. Heat leakage between compression and expansion spaces and heat transfer to the environment are negligible.
12. Leakage of gas to the outside of the engine is assumed to be zero.

In the Adiabatic model presented by Urieli and Berchowitz [17], the Stirling engine was divided into five compartments, as illustrated in Fig. 1. In this figure, single suffixes c, k, r, h, e represent five compartments including compression space, Cooler (cooler), regenerator, heater and expansion space, respectively. Double suffixes (ck, kr, rh, he) are dedicated to four interfaces between the compartments.

The major governing equations were derived based on state equations of the ideal gas and conservation equations of mass and energy in each compartment. By developing the governing equations for five compartments of the Stirling engine, a system of ordinary differential equations was obtained. Table 1 indicates system of ordinary differential equation, mass flow equations between compartments and conditional temperature of the entire Stirling engine. In the equations of Table 1, the variables including V_c , V_e ,

Table 1

System of ordinary differential equations in the Adiabatic analysis [17].

$P = \frac{MR}{V_c + V_k + V_r + V_h + V_e}$	Pressure
$dP = \frac{-\gamma P \left(\frac{dV_c + dV_e}{V_c + V_k + V_r + V_h + V_e} \right)}{\frac{V_k}{T_{ck}} + \gamma \left(\frac{V_k}{T_k} + \frac{V_h}{T_h} \right) + \frac{V_e}{T_{he}}}$	Pressure variation
$m_i = \frac{PV_i}{RT_i}, \quad i = c, k, r, h, e$	Mass
$dm_e = \frac{PdV_e + V_e \frac{dP}{P}}{RT_{he}}$	Mass variation
$dm_c = \frac{PdV_c + V_c \frac{dP}{P}}{RT_{ck}}$	
$dm_i = m_i \frac{dP}{P}, \quad i = k, r, h$	
$\dot{m}_{ck} = -dm_c$	Mass flow
$\dot{m}_{kr} = \dot{m}_{ck} - dm_k$	
$\dot{m}_{rh} = \dot{m}_{he} + dm_h$	
if $\dot{m}_{ck} > 0$ then $T_{ck} = T_c$ else $T_{ck} = T_k$	Conditional temperature
if $\dot{m}_{he} > 0$ then $T_{he} = T_h$ else $T_{he} = T_e$	
$dT_e = T_e \left(\frac{dP}{P} + \frac{dV_e}{V_e} - \frac{dm_e}{m_e} \right)$	Temperature variation
$dT_c = T_c \left(\frac{dP}{P} + \frac{dV_c}{V_c} - \frac{dm_c}{m_c} \right)$	
$dQ_k = \frac{V_k c_p dP}{R} + c_p (T_{ck} dm_c - T_{kr} (dm_c + dm_k))$	Heat output of cooler
$dQ_r = \frac{V_r c_p dP}{R} + c_p (T_{kr} (dm_c + dm_k) - T_{rh} (dm_c + dm_k + dm_r))$	Heat receiving and rejecting from the regenerator
$dQ_h = \frac{V_h c_p dP}{R} + c_p (T_{rh} (dm_c + dm_k + dm_r) - T_{he} (-dm_e))$	Heat input to heater
$dW_e = PdV_e$	Work in expansion process
$dW_c = PdV_c$	Work in compression process

dV_c and dV_e are obtained based on the engine configuration as analytic functions of the crank angle (θ). V_k , V_r , V_h are dead volumes of the engine in cooler, regenerator and heater, respectively. These parameters are determined based on geometric specifications of heat exchangers and regenerator.

Classical fourth-order Runge-Kutta method is usually used to solve systems of boundary-value ordinary differential equation.

In the ideal adiabatic model, regenerator is assumed to work in an ideal regeneration process. In the ideal regenerative process, heat absorption of working fluid in the heating mode is totally transferred to the fluid in the cooling mode. Furthermore, pressure losses in the regenerator and heat exchanger are neglected. Simple analysis or actual Adiabatic analysis is developed to include ineffectiveness of a real Stirling engine into the thermal model [17]. In Simple analysis, imperfect regeneration analysis is performed to determine effect of imperfect regeneration process on the magnitude of gas temperature. Moreover, power loss due to pressure drops in the regenerator and heat exchangers is determined and subtracted from the power output obtained from the ideal Adiabatic analysis.

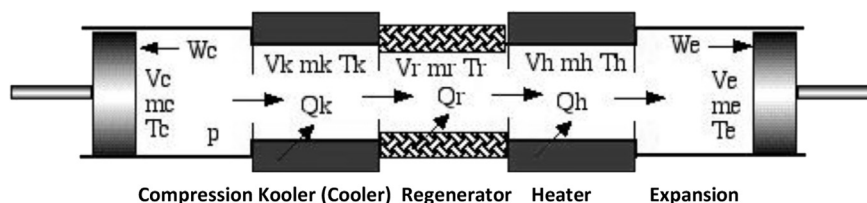


Fig. 1. Schematic model for five compartments of the Stirling engine [15].

2.2. Simple-II: new thermal model

Simple-II works based on similar concepts to those of Simple analysis. In this model, first, effects of shuttle heat transfer loss and gas leakage to crank case were implemented in a differential form into the ideal adiabatic analysis. In this regard, the system of ordinary differential equations of adiabatic analysis [17] was modified to include shuttle effect heat transfer loss and gas leakage and a new system of equations was obtained. In the next step, in a similar manner to the Simple analysis, results of the previous step were modified in Simple-II thermal model to include effects of pressure loss in the regenerator, heater and cooler, pressure loss due to finite motion of piston, power loss due to mechanical friction, conduction heat loss in the regenerator wall and non-ideal heat transfer of heat exchangers. It should be noted that the original Simple analysis [17] contained only effect of non-ideal heat transfer and pressure drop in heat exchangers. In the Simple-II thermal model presented in this paper, other loss effects (shuttle effect, gas leakage, pressure loss due to finite motion of piston, power loss due to the mechanical friction, conduction heat loss in the regenerator wall) were also integrated into the model for the first time. In summary, it can be said that, in the developed new thermal model (Simple-II), loss effects of Stirling engines were divided into three categories. One part of loss mechanisms including gas leakage and shuttle effect was considered in the basic ordinary differential equations. The second category of losses was non-ideal heat transfer and pressure drops in heat exchangers that were differentially evaluated in combination with the first part and used to correct temperature of engine spaces. The third category of losses was pressure loss due to finite motion of piston, mechanical friction and longitudinal heat conduction between heater and cooler through the regenerator wall which were calculated as separate loss terms which did not affect temperature distribution of engine spaces. Fig. 2 summarizes the three categories of loss mechanisms in an Stirling engine.

In Simple-II thermal model, the list of assumptions of original Simple analysis [17] was modified and assumptions #10, #11 and #12 as cited in Section 2.1 of this paper for original Simple analysis were omitted in developing the Simple-II thermal model.

2.2.1. Modifying ideal adiabatic analysis

In the new model (Simple-II) in a similar manner to the ideal Adiabatic analysis, the engine was divided into five compartments of heater, Kooler (cooler), regenerator, expansion and compression compartments, as in Fig. 1. Also, heat transfer in the heat exchangers was unsteady and temperature in the heater and cooler varied with respect to the crank angle. In developing the system of

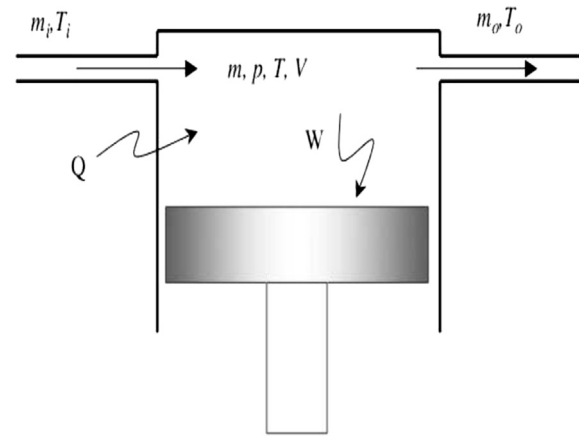


Fig. 3. Scheme of a sample engine compartment.

differential equations for governing equations, derivation of each parameter with respect to crank angle (θ) was denoted by d ; therefore, $d \equiv d/d\theta$ hereinafter. In the basic analysis of Stirling engines in the Simple-II model, the energy conservation law for each compartment of the engine (as in Fig. 1) was developed in a differential form to include effect of the shuttle heat loss. For each compartment, a hypothetical control volume which is illustrated in Fig. 3 was considered. For a control volume as illustrated in Fig. 3, equation of the first law of thermodynamics in a differential form was:

$$dQ - dQ_{\text{shuttle}} + (\dot{m}_i c_p T_i - \dot{m}_o c_p T_o) = dW + c_v d(mT) \quad (1)$$

where dQ is heat absorption or rejection from (or into) the working fluid via the external resources and dW shows the net output work. dQ_{shuttle} denotes heat loss due to shuttle conduction by the displacer. This is the new term that was added to the energy conservation equation in the new model (Simple-II) presented in this paper. dQ_{shuttle} could be obtained as follows [23]:

$$dQ_{\text{shuttle}} = \frac{\pi S^2 k_g D_d}{8JL_d} (T_e - T_c) \quad (2)$$

where S , k_g , D_d , J , L_d , T_e and T_c are stork, thermal conductivity of gas, diameter of displacer, gap between displacer and cylinder, displacer length, gas temperature in expansion space and gas temperature in compression space, respectively.

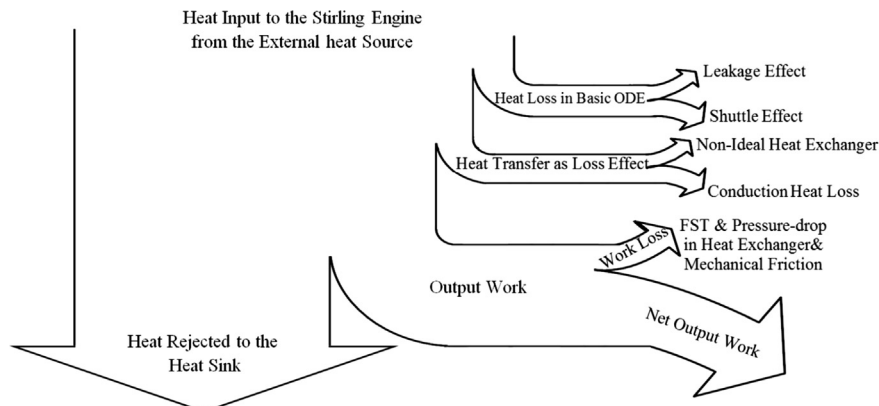


Fig. 2. Various loss effects of Stirling engines.

The third term on the left hand side of Eq. (1) determines the net absorbed enthalpy of the working fluid in each compartment of the engine due to in/out flows. The second term in the right hand of the energy conservation equation shows change of differential internal energy of the working fluid.

Equation of state of the ideal gas for the working fluid can be written as:

$$PV = mRT \quad (3a)$$

Differentiating Eq. (3a) leads to:

$$\frac{dP}{P} + \frac{dV}{V} = \frac{dm}{m} + \frac{dT}{T} \quad (3b)$$

Total mass of the working fluid (M) is:

$$M = m_c + m_k + m_r + m_h + m_e - m_{leak} \quad (4)$$

where m_{leak} denotes mass leakage to crank case due to the leakage effect. This term is a new factor that was added to the mass conservation law of the original adiabatic analysis in Simple-II model. The leakage to the crank case was calculated as follows [17]:

$$m_{leak} = \pi D \frac{P + P_{buffer}}{4RT_g} \left(u_p J - \frac{J^3}{6\mu} \frac{P - P_{buffer}}{L} \right) \quad (5)$$

where J , D , T_g , L , u , μ , P and P_{buffer} are annular gap between piston and cylinder, piston diameter, gas temperature, piston length, piston linear velocity, viscosity of working flow, pressure of working space and pressure of buffer space (P_{buffer} was assumed 100 kPa in this paper).

With these definitions, Eq. (4) can be rewritten in the following form:

$$M = \left\{ \frac{P \left(\frac{V_c}{T_c} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_e} \right)}{R} \right\} - \left\{ \pi D \frac{P + P_{buffer}}{4RT_g} \left(u_p J - \frac{J^3}{6\mu} \frac{P - P_{buffer}}{L} \right) \right\} \quad (6)$$

The mass equation in a differential form was obtained by differentiating Eq. (4) as follows:

$$dm_c + dm_k + dm_r + dm_h + dm_e - dm_{leak} = 0 \quad (7)$$

As for the heater, cooler and regenerator, the following was assumed: $dV/V \cong 0$.

Therefore, differential mass equation (Eq. (3b)) can be written as follows:

$$\frac{dP}{P} \Big|_{k,h,r} = \frac{dm}{m} \Big|_{k,h,r} \quad (8a)$$

Therefore,

$$dm_{k,h,r} = \frac{m dP}{P} \Big|_{k,h,r} = \frac{V dP}{RT} \Big|_{k,h,r} \quad (8b)$$

Substitution of Eq. (8b) for Eq. (7) led to the following expression:

$$dm_c + dm_e - dm_{leak} + \frac{dP \left(\frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} \right)}{R} = 0 \quad (9)$$

Assuming the adiabatic compression process, the energy conservation equation (Eq. (1)) for compression space can be rewritten as follows:

$$-dQ_{shuttle} - m_{ck} c_p T_{ck} = dW_c + c_v d(m_c T_c) \quad (10)$$

The expression for variation of compression gas mass (dm_c) was obtained using state equation of ideal gases and some intermediate calculations as follows:

$$dQ_{shuttle} + c_p T_{ck} dm_c = PdV_c + c_v d(m_c T_c)$$

$$dQ_{shuttle} + c_p T_{ck} dm_c = PdV_c + c_v d\left(\frac{PV_c}{R}\right)$$

$$dQ_{shuttle} + c_p T_{ck} dm_c = PdV_c + \frac{c_v P}{R} dV_c + \frac{c_v V_c}{R} dP$$

$$dQ_{shuttle} + c_p T_{ck} dm_c = \frac{c_p}{R} PdV_c + \frac{c_v}{R} V_c dP$$

$$\frac{dQ_{shuttle}}{c_p} + T_{ck} dm_c = \frac{PdV_c}{R} + \frac{c_v}{R c_p} V_c dP$$

$$\frac{dQ_{shuttle}}{T_{ck} c_p} + dm_c = \frac{PdV_c}{RT_{ck}} + \frac{1}{RT_{ck}} \frac{V_c}{\gamma} dP$$

$$dm_c = \frac{PdV_c + \frac{V_c}{\gamma} dP}{RT_{ck}} - \frac{dQ_{shuttle}}{c_p T_{ck}} \quad (11)$$

In a similar manner for the expansion space, the following can be obtained:

$$dm_e = \frac{PdV_e + \frac{V_e}{\gamma} dP}{RT_{he}} - \frac{dQ_{shuttle}}{c_p T_{he}} \quad (12)$$

Substituting dm_c and dm_e obtained from Eqs. (11) and (12) for Eq. (9) led to the following expression of differential pressure in the working space:

$$dP = \frac{-P \left(\frac{dV_c}{T_{ck}} + \frac{dV_e}{T_{he}} \right) + \frac{dQ_{shuttle} R}{c_p} \left(\frac{T_{he} - T_{ck}}{T_{he} T_{ck}} \right) + R dm_{leak}}{\frac{V_c}{\gamma T_{ck}} + \left(\frac{V_k}{T_k} + \frac{V_h}{T_h} + \frac{V_r}{T_r} \right) + \frac{V_e}{\gamma T_{he}}} \quad (13)$$

Moreover, the differential temperature equations were obtained as follows (see Eq. (3b)):

$$dT_c = T_c \left(\frac{dP}{P} + \frac{dV_c}{V_c} - \frac{dm_c}{m_c} \right) \quad (14)$$

$$dT_e = T_e \left(\frac{dP}{P} + \frac{dV_e}{V_e} - \frac{dm_e}{m_e} \right)$$

Heat transfer rate in the cooler, heater and regenerator could be evaluated from the energy conservation equation as follows:

$$dQ_k = \frac{V_k dP c_v}{R} - c_p (T_{ck} \dot{m}_{ck} - T_{kr} \dot{m}_{kr}) \quad (15)$$

$$dQ_h = \frac{V_h dP c_v}{R} - c_p (T_{he} \dot{m}_{he} - T_{hr} \dot{m}_{hr}) \quad (16)$$

$$dQ_r = \frac{V_r dP c_v}{R} - c_p (T_{kr} \dot{m}_{kr} - T_{rh} \dot{m}_{rh}) \quad (17)$$

The output work performed by the power piston can be calculated as follows:

$$dW_c = PdV_c \quad (18)$$

$$dW_e = PdV_e \quad (19)$$

2.2.2. Implication of other loss effects considered in Simple-II model

System of ordinary differential equations obtained in Section 2.2.1 was solved using the fourth-order Ruge-Kutta method in a similar manner that differential equations were solved in original ideal adiabatic analysis [17]. The difference between the present model and original ideal adiabatic model was that, in the present model, shuttle effect and gas leakage terms were included in differential equations. Effect of the losses including non-ideal heat transfer in heat exchangers, pressure drops in heat exchanger, pressure loss due to finite motion of piston, power loss due to mechanical friction and conduction heat loss in the regenerator wall was divided into two groups. The first group including non-ideal heat transfer and pressure drops in heat exchangers was differentially evaluated in combination with the first part and used to correct temperature of the engine spaces. The next group of losses including pressure loss due to finite motion of the piston, mechanical friction and longitudinal heat conduction between the heater and cooler through the regenerator wall was calculated as separate loss terms that did not affect temperature distribution of engine spaces; these loss effects were subtracted from the results obtained from solution of differential equation. In the following sections, principles or evaluation of these losses are presented.

2.2.2.1. Non-ideal heat transfer effect. In Stirling engines, heat is transferred to the working fluid in the heater and cooler. Then, it is absorbed and recovered in a regenerative heat exchanger, known as the regenerator. For perfect heat recovery, the regenerator is designed to be very compact and efficient. The main heat transfer mechanism in the regenerator is forced convection heat transfer. Special structure of the regenerator increases frictional pressure loss (pumping loss).

Performance of Stirling engines for heat recovery is evaluated by regenerator effectiveness. Effectiveness is defined as a ratio of real enthalpy change to maximum enthalpy change of working fluid in the regenerator. Regenerator effectiveness changes from 0.0 (for no heat recovery) to 1.0 (for complete heat recovery). When effectiveness is less than unity, the output gas temperature from the regenerator is less than temperature of the heater. This heat should be supplied by an external heat source, which causes efficiency reduction. These definitions can be written for the heater and cooler, respectively, as follows:

$$Q_h = Q_{h_{ideal}} + Q_{r_{loss}} = Q_{h_{ideal}} + Q_{r_{ideal}}(1 - \varepsilon) \quad (20)$$

$$Q_k = Q_{k_{ideal}} - Q_{r_{loss}} = Q_{k_{ideal}} - Q_{r_{ideal}}(1 - \varepsilon) \quad (21)$$

where $Q_{r_{loss}}$ is heat loss in the regenerator, Q_h is heat addition to the working fluid in the heater, Q_k is heat rejection to the cooler and ε is the regenerator effectiveness.

Regenerator effectiveness could be evaluated as a function of the input and output temperatures by assuming a linear profile in the regenerator [17]:

$$\varepsilon = \frac{1}{1 + \frac{\delta T}{T_{hi} - T_{ho}}} \quad (22)$$

where δT is temperature difference between the hot and cold streams in the regenerator and T_{hi} and T_{ho} are hot stream temperatures at the input and output of the regenerator.

The energy balance equation for the hot stream ($\dot{m}c_p(T_{hi} - T_{ho}) = hA_{wg}\delta T$) leads to the following expression for regenerator effectiveness:

$$\varepsilon = \frac{1}{1 + \frac{\dot{m}c_p}{hA_{wg}}} = \frac{1}{1 + \frac{1}{NTU}} = \frac{NTU}{1 + NTU} \quad (23)$$

Convection heat transfer coefficient (h) can be calculated as follows [32]:

$$St = 0.023Re^{-0.2}Pr^{-0.6} \quad (24)$$

$$NTU = \frac{StA_{wg}}{A} \quad (25)$$

In Eqs. (23)–(25), NTU , St , A_{wg} , A , h , Re and Pr are the number of transfer unit, Stanton number, wet area of gas in the heat exchanger, cross-section area of heat exchanger, convective heat transfer coefficient, Reynolds number and Prandtl number, respectively. In the evaluation of the Reynolds number, hydraulic diameter of the regenerator mesh is obtained as [32]:

$$D_{H,r} = \frac{4\Pi}{\Phi(1 - \Pi)} \quad (26)$$

$$\Phi = \frac{A_{wg}}{V_{mesh}} \quad (27)$$

where V_{mesh} , Π and Φ are volume of wires, porosity and shape factor of the regenerator mesh, respectively.

Because of non-ideal heater and cooler, the working fluid temperature in these two heat exchangers are lower and higher than the wall temperature, respectively. By calculating the heat loss in non-ideal regenerator and heat transfer calculation, the wall temperature in cooler and heater can be calculated as follows:

$$Q_h = Q_{h_{ideal}} + Q_{r_{loss}} = \frac{h_h A_{wh}(T_{wh} - T_h)}{f} \quad (28)$$

where f , T_{wh} , h_h , A_{wh} and Q_h are engine rotation frequency, wall temperature in the heater, convective heat transfer of the working fluid in the heater, heat transfer area of the heater and rate of heat transfer in the heater, respectively. Therefore, the heater temperature is corrected from the value of heater wall temperature as follows:

$$T_h = T_{wh} - \frac{Q_h f}{h_h A_{wh}} \quad (29)$$

In a similar manner, the cooler temperature is corrected as:

$$T_k = T_{wk} - \frac{Q_k f}{h_k A_{wk}} \quad (30)$$

In Eqs. (29) and (30), heat transfer coefficient is obtained using the following correlation [17]:

$$h_{k,h} = \frac{0.0791 \mu \cdot c_p \cdot Re_m^{0.75}}{2D_H \cdot Pr} \quad (31)$$

where Re_m and D_H are Reynolds number and hydraulic diameter of heater or cooler.

Corrected values of heater and cooler temperatures give feedback to the solver of the system of differential equations (cited in Section 2.2.1) until the convergence criteria for the magnitude of gas temperatures is satisfied.

2.2.2.2. Pressure drop effect. Pressure drops in heat exchangers due to the fluid friction cause power loss in Stirling engines. The pressure drop in heat exchangers should be calculated in order to calculate the power loss due to pressure losses. The pressure drops in heat exchangers can be obtained as:

$$\Delta P = -\frac{2f_{Re}\mu uV}{d^2A} \quad (32)$$

where d is hydraulic diameter, u is velocity, V is volume, A is flow cross-section area and f_{Re} is a Reynolds friction factor. f_{Re} is defined as follows [17]:

$$f_{Re} = \begin{cases} 16 & Re < 2000 \\ 7.343 \times 10^{-4} Re^{1.3142} & 2000 < Re < 4000 \\ 0.0791 Re^{0.75} & Re > 4000 \end{cases} \quad (33)$$

The net power loss due to pressure drop in heat exchangers of the Stirling engine is evaluated as follows:

$$W_{loss} = \int \sum \Delta P dV_e \quad (34)$$

2.2.2.3. Effect of finite speed of piston and mechanical friction. Based on finite speed thermodynamic principle, the pressure in compression and expansion pistons differs from the calculated instantaneous pressure of expansion and compression spaces. It was demonstrated that pressure over the piston in expansion space was less than the mean pressure of the cylinder in the expansion process. Similarly, the pressure over the piston surface in compression process is greater than mean pressure of the cylinder. Therefore, the work obtained in expansion process is less than that of theoretical expansion. Similarly, the actual compression work is greater than the theoretical work of compression which is calculated in classical thermodynamics. Therefore, output power of the real engine decreases from the corresponding value calculated using classical thermodynamics. The pressure loss caused by finite speed effect and mechanical friction can be calculated using Eqs. (35) and (36) as follows [33]:

$$P_{cylinder-corrected} = P_{cylinder} \left(1 \pm \frac{au_p}{c} \pm \frac{\Delta P_f}{P_{cylinder}} \right) \quad (35)$$

$$\Delta P_{finite\ speed} = \pm \frac{au_p}{c} \pm \frac{\Delta P_f}{P_{cylinder}} \quad (36)$$

Finite speed movement of the piston that generates pressure waves in working spaces causes finite speed pressure losses. In the above Eqs. (35) and (36), sign (+) is used for compression space and sign (−) are used for expansion space. Also, u_p is the piston velocity, c is average speed of molecules and ΔP_f is piston-cylinder friction loss. In Eqs. (35) or (36), ΔP_f , a and c are calculated as follows [34]:

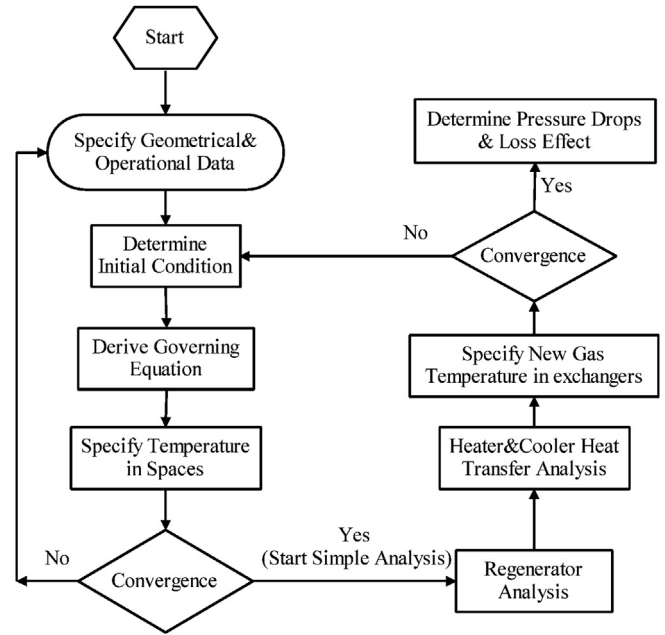


Fig. 4. Flow-chart of the Simple-II thermal model.

Table 2

System of ordinary differential equations obtained in the new (Simple-II) thermal model.

$\left\{ \frac{P \left(\frac{V_c}{T_c} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_e} \right)}{R} \right\} - \left\{ \pi D \frac{P + P_{puffer}}{4RT_g} \left(u_p J - \frac{J^2}{6\mu} \frac{P - P_{puffer}}{L} \right) \right\} = M$	Pressure
$dP = \frac{-P \left(\frac{dV_c}{V_c} + \frac{dV_e}{V_e} \right) + \frac{dQ_{shuttle}}{c_p} \left(\frac{T_{he} - T_{ck}}{T_{he} T_{ck}} \right) + R dm_{leak}}{\frac{V_c}{T_{ck}} + \left(\frac{V_k}{T_k} + \frac{V_h}{T_h} + \frac{V_r}{T_r} \right) + \frac{V_e}{T_{he}}}$	Pressure variation
$m_i = \frac{P V_i}{R T_i}, \quad i = c, k, r, h, e$	Mass
$m_{leak} = \pi D \frac{P + P_{puffer}}{4RT_g} \left(u_p J - \frac{J^2}{6\mu} \frac{P - P_{puffer}}{L} \right)$	
$dm_e = \frac{P dV_e + \frac{V_e}{T_e} dP}{RT_{he}} - \frac{dQ_{shuttle}}{c_p T_{he}}$	Mass variation
$dm_c = \frac{P dV_c + \frac{V_c}{T_c} dP}{RT_{ck}} - \frac{dQ_{shuttle}}{c_p T_{ck}}$	
$dm_i = m_i \frac{dP}{P}, \quad i = k, r, h$	
$\dot{m}_{ck} = -dm_c$	Mass flow
$\dot{m}_{kr} = \dot{m}_{ck} - dm_k$	
$\dot{m}_{rh} = \dot{m}_{he} - dm_h$	
for $\dot{m}_{ck} > 0$, $T_{ck} = T_c$ else $T_{ck} = T_k$ for $\dot{m}_{he} > 0$, $T_{he} = T_h$ else $T_{he} = T_e$	Conditional temperature
$dT_e = T_e \left(\frac{dP}{P} + \frac{dV_e}{V_e} - \frac{dm_e}{m_e} \right)$	Temperature variation
$dT_c = T_c \left(\frac{dP}{P} + \frac{dV_c}{V_c} - \frac{dm_c}{m_c} \right)$	
$dQ_k = \frac{V_k dP_{ck}}{R} - c_p (T_{ck} \dot{m}_{ck} - T_{kr} \dot{m}_{kr})$	Heat output of cooler
$dQ_r = \frac{V_r dP_{cr}}{R} - c_p (T_{kr} \dot{m}_{kr} - T_{rh} \dot{m}_{rh})$	Heat receiving and rejecting from the regenerator
$dQ_h = \frac{V_h dP_{ch}}{R} - c_p (T_{he} \dot{m}_{he} - T_{he} \dot{m}_{he})$	Heat input to heater
$dW_e = P dV_e$	Work in expansion process
$dW_c = P dV_c$	Work in compression process

Table 3
Design specifications of the GPU-3 Stirling engine [26].

General information	Working fluid	Helium	Regenerator	Regenerator internal diameter	22.6 mm
	Piston stroke	31.2 mm		Regenerator length	22.6 mm
	Internal diameter of cylinder	69.9 mm		Number of regenerator	8
	Frequency	41.7		Material	Stainless steel
	Mean pressure	4.13 MPa		Diameter of regenerator tube	0.04 mm
	Phase angle	90		Porosity	0.697
	Heater temperature	977 K	Cooler	Average of tube length	46.1 mm
	Cooler temperature	288 K			
	Number of Cylinder	1		Internal diameter of tube	1.09 mm
	Average tube length	245.3 mm		External diameter of tube	1.59 mm
Heater	Tube external diameter	4.83 mm		Number of tubes	312
	Number of tubes	40	Volume	Swept Vol. (expansion/compression)	120.88/113.14 mm ³
	Tube internal diameter	3.02 mm			
	Tube length (cylinder side)	116.4 mm		Clearance Vol. (expansion/compression)	30.52/28.68 mm ³
	Tube length (regenerator side)	128.9 mm		Dead Vol. (heater/cooler/regenerator)	70.88/13.8/50.55 mm ³

$$\Delta P_f = 0.97 + 0.15 \frac{N_r}{1000} \quad (37a)$$

$$a = \sqrt{3\gamma} \quad (37b)$$

$$c = \sqrt{3RT} \quad (37c)$$

where N_r is angular rotational speed of the engine in rpm.

Finally, the work loss due to finite speed of the piston motion and mechanical friction can be calculated as follows:

$$W_{\text{loss-finite speed}} = \int \left(\pm \frac{a u_p}{c} \pm \frac{\Delta P_f}{P_{\text{cylinder}}} \right) dV \quad (38)$$

2.2.2.4. Effect of heat loss from the regenerator wall. Heat conduction leakage between the heater and cooler through the regenerator wall is evaluated using Eq. (39) [35]:

$$Q_{\text{reg-leak}} = R_{\text{cond}}(T_{\text{wh}} - T_{\text{wk}}) \quad (39)$$

where R_{cond} , T_{wh} and T_{wk} are conduction resistance of the regenerator wall and wall temperatures of the heater and cooler, respectively.

3. Solution method

Solution algorithm of Simple-II thermal model is illustrated in Fig. 4.

As is clear in Fig. 4, in the first step of the calculation, system of ordinary differential equation obtained in Section 2.2.1 was numerically solved. This step was quite similar to the original ideal adiabatic analysis, except that system of differential equations was corrected to include effect of shuttle heat transfer loss and mass

leakage. In a similar manner to Table 1, system of ordinary differential equations is summarized in Table 2.

In the system of equations, variables including V_c , V_e , dV_c and dV_e are obtained based on the engine configuration as analytic functions of the crank angle θ . V_k , V_r and V_h are dead volumes of the engine in cooler, regenerator and heater, determined based on geometrical specifications of heat exchangers and regenerator. Regenerator temperature is considered as mean effective temperature of heater and cooler [17]. Except for the aforementioned constants, it is required to solve 16 ordinary differential equations including 24 variables over a complete cycle (see Table 2), among which seven derivatives (T_c , $T_e Q_k$, Q_r , Q_h , W_c and W_e) are numerically integrated using the fourth-order Runge-Kutta method [17] and eleven variables and derivatives including W , p , Q_{shuttle} , V_c , V_e , m_c , m_k , m_r , m_h , m_e and m_{leak} are analytically determined. Moreover, there are six boundary conditions and mass flow rate equations the same as those indicated in Table 1 for original ideal adiabatic analysis. In this case, the equations were initial-value problems. Initial estimation of the working fluid mass was performed using Schmidt model [14]. Initial gas temperatures in compression and expansion cylinders were estimated to be equal to the cooler and heater temperatures, respectively. Then, numerical solutions of differential equations were performed for several times until the steady-state condition was obtained, in which the magnitudes of temperatures at the end of cycle converged to the corresponding temperatures at the beginning of the cycle. In other words, magnitude of temperatures at the start of the cycle ($\theta = 0$) had to be equal to the corresponding values at the end of the cycle ($\theta = \pi$). Hence, the calculations were repeated until the same gas temperatures were obtained for $\theta = 0$ and $\theta = \pi$.

As is clear in Fig. 4, in the next step, calculation was entered into the Simple analysis section, in which the non-ideal heat transfer effect was taken into account to correct magnitude of cooler and heater temperatures using Eqs. (29) and (30). The corrected magnitudes of cooler and heater temperatures gave feedback to the numerical solution section until convergence for the magnitude of

Table 4
Comparison between the current and other models.

Model	Ideal adiabatic model		Urieli & Berchowitz Model (Simple Analysis) [12]		Timoumi model [19]		Current study (Simple-II model)		Experimental result [31]
	Actual data	Error ^a (%)	Actual data	Error ^a (%)	Actual data	Error ^a (%)	Actual data	Error ^a (%)	
Efficiency (%)	62.3	+40.0	52.5	+31.2	38.5	+17.2	28.4	+7.1	21.3
Power (kW)	8.30	+176.7	6.70	+123.3	4.27 ^b	+42.3	3.62	+20.7	3.0

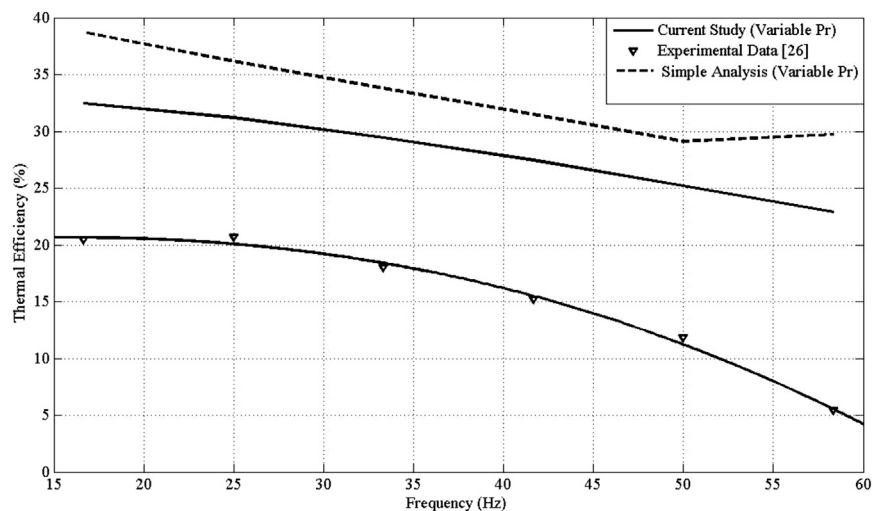
^a Error in thermal efficiency was calculated as a difference between the actual value and experimental data.

^b Hysteresis loss was considered (without the effect of hysteresis power was obtained as 5.90 kW in Ref. [19]).

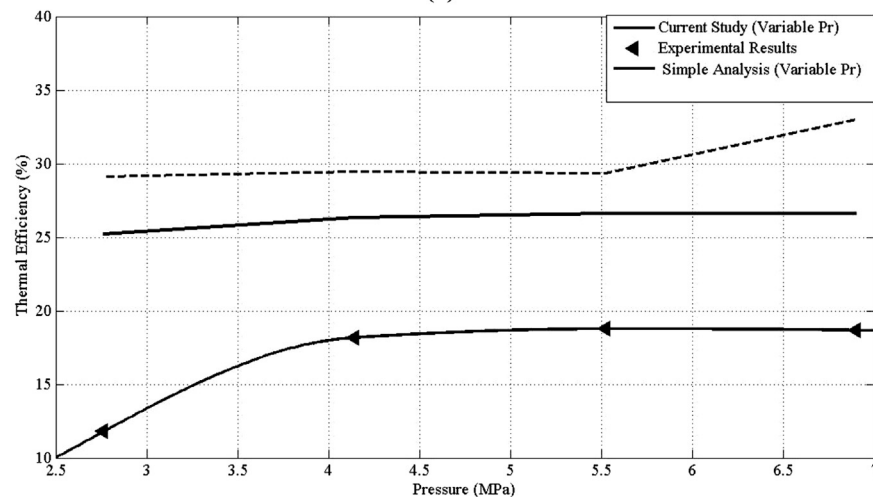
Table 5

Comparison between the current results and experimental results at various engine operation modes.

Frequency (Hz)	Mean effective pressure (MPa)	Thermal efficiency predicted by the simple analysis (variable Pr)			Thermal efficiency predicted by the current study (Simple-II model) (variable Pr)			Experimental efficiency [26]
		Actual value (%)	Error (as differences) (%)	Average error (as difference) (%)	Actual value (%)	Error (as differences) (%)	Average error (as difference) (%)	
16.67	2.76	38.72	18.22	17.90	32.48	11.98	12.85	20.50
25.00	2.76	36.16	15.46		31.21	10.51		20.70
33.33	2.76	33.79	15.79		29.45	11.45		18.00
41.67	2.76	31.48	16.28		27.45	12.25		15.20
50.00	2.76	29.12	17.32		25.21	13.41		11.80
58.33	2.76	29.74	24.34	11.46	22.89	17.49	8.28	5.40
25.00	4.14	35.65	10.85		32.29	7.49		24.80
33.33	4.14	33.52	9.62		30.40	6.50		23.90
41.67	4.14	31.48	10.18		28.39	7.09		21.30
50.00	4.14	29.45	11.25		26.33	8.13		18.20
58.33	4.14	27.40	15.40		24.21	12.21	8.11	12.00
41.67	5.52	31.20	8.70	10.82	28.59	6.09		22.50
50.00	5.52	29.33	10.53		26.62	7.82		18.80
58.33	5.52	27.44	13.24		24.62	10.42		14.20
50.00	6.90	29.07	10.37	11.73	26.61	7.91	9.19	18.70
58.33	6.90	27.29	13.09		24.67	10.47		14.20



(a)



(b)

Fig. 5. Comparison of thermal efficiency between original Simple analysis and the current study's (Simple-II thermal model) at (a) various frequencies and $P_{\text{mean}} = 2.76$ MPa; (b) various mean effective pressures and $fr = 50$ Hz.

temperatures was obtained. Finally, magnitude of output power was corrected for power losses due to pressure drops in heat exchangers, mechanical friction and finite motion of the piston. In addition, the magnitude of heat transfer in the heater was corrected for including effect of conductive heat loss through the regenerator wall.

4. Case study

Evaluation of the developed thermal model was performed by considering the GPU-3 Stirling engine as a case study. The GPU-3 Stirling engine was built in research laboratories of General Motors as a 3.0 kW Stirling engine and was tested in the NASA Lewis Research Center. This engine is a beta-type engine with the rhombic drive and compact heat exchangers and regenerator with a woven mesh. Design specifications of the GPU-3 Stirling engine are indicated in Table 3.

5. Results and discussion

5.1. Model verification and error analysis

Comparison between the results of the current new thermal model (Simple-II) and corresponding results obtained from previous models and experimental data were indicated in Table 4. Comparisons of the current results with experimental results showed that Simple-II model had better agreement with the experimental results in comparison to the previous thermal models so that errors in the prediction of the output power and thermal efficiency were reduced from 42.3% and 17.2% (as difference) in the best previous model [19] to 20.7% and 7.1% (as difference) in the present model, respectively. Therefore, the current thermal model improved errors on the output power and thermal efficiency by about 21.6% and 10.1%, respectively, in comparison to the best of previous thermal models [19].

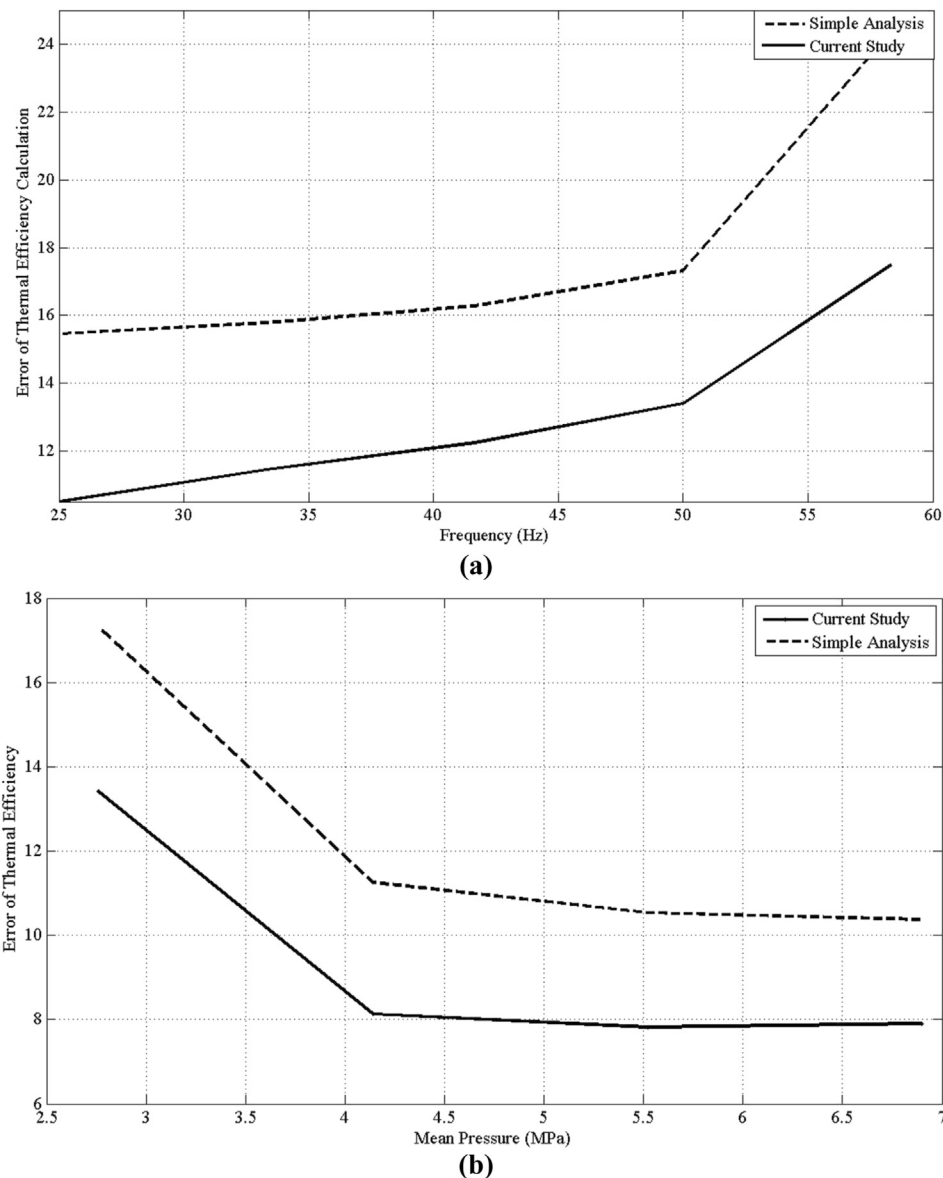


Fig. 6. Comparison of thermal efficiency error between original Simple analysis and the current study's (Simple-II thermal model) at (a) various frequencies and $P_{\text{mean}} = 2.76$ MPa; (b) various mean effective pressures and $f = 50$ Hz.

Similarly, Table 5 compares Simple-II thermal model and Simple model in terms of predicting thermal efficiency at various engine rotation speeds (rotation frequencies) and mean effective pressures. This table indicates that, in general, error in prediction of thermal efficiency for both models decreased as mean effective pressure of the engine increased. Similarly, it can be said that the error of both models was generally higher when rotation frequency of the engine increased or rotation speed was very low. Table 5 re-confirms that the presented thermal model predicted thermal efficiency with lower error in comparison to the original Simple analysis. Based on Table 5, thermal efficiency of the Stirling engine reduced with increasing rotation frequency or mean effective pressure. Based on Table 5, thermal efficiency of the Stirling engine reduced with increasing rotation frequency or mean effective pressure.

For a better insight into effect on rotation frequency and mean pressure on thermal efficiency of the engine, Fig. 5a and b is presented here, respectively. Effect of frequency on efficiency of Stirling engine at constant pressure (2.76 MPa) is illustrated in Fig. 5a, which confirms that thermal efficiency of the engine reduced when rotation frequency increased. It could be found in Fig. 5a that both models had reasonable deviation from the experimental data; nevertheless, the proposed Simple-II thermal model not only had less deviation but also showed a better matching variation trend to the actual experimental trend. Based on the original Simple analysis, thermal efficiency decreased linearly when rotation frequency decreased to 50 Hz and increased linear again as rotation frequency increased to above 50 Hz. However, as is clear in Fig. 5a, experimental data demonstrated that thermal efficiency of the engine decreased for the entire increasing range of rotation speed. In this regard, the variation trend of thermal efficiency with rotation speed in Simple-II thermal model was quite similar to that of the experimental data.

Effect of effective mean pressure on thermal efficiency of Stirling engine at constant rotation frequency ($f_r = 50$ Hz) is depicted in Fig. 5b. It is clear that Simple-II model predicted thermal efficiency with less deviation from real data in comparison to the original Simple model. Based on the original Simple model, thermal efficiency remained almost constant when the mean pressure was increased to 5.5 MPa and linearly increased with a higher rate when mean pressure increased to above 5.5 MPa. Based on the present model, a small linear increasing rate of thermal efficiency was

observed when mean pressure was increased over the entire variation range of the mean pressure. Actual data showed that, over 4.0 MPa, increasing rate of thermal efficiency was similar to what predicted by Simple-II model; but, when mean pressure was less than 4.0 MPa, increasing rate of thermal efficiency of the real data was higher than what was predicted by both models.

Fig. 6a and b illustrates variation in errors of two models in terms of predicting thermal efficiency against variation of rotation frequency and mean pressure, respectively. Based on Fig. 6a, errors in the prediction of thermal efficiency for both models increased linearly when frequency increased from 25 Hz to 50 Hz; rate of increase in error increased drastically when frequency increased to above 50 Hz. The linear increasing rate of error for Simple-II model was a little higher than that of Simple analysis when frequency was smaller than 50 Hz. When frequency was over 50 Hz, linear increasing rate of Simple-II model was lower than the corresponding error predicted by the Simple analysis. As is clear, absolute value of error obtained in Simple-II model was lower than the corresponding error of Simple analysis for all the values related to engine frequency. Fig. 6b indicates that, for both models, magnitude of error in prediction of thermal efficiency (as difference) linearly decreased when mean pressure increased up to ≈ 4.2 MPa. Over 4.2 MPa, variation errors of both models were non-significant as mean pressure varied. Variation rate of errors for both models was almost the same; however, in all case, Simple-II model had less error than Simple analysis.

5.2. Performance prediction of the case studied Stirling engine

P – V diagram of the compression and expansion spaces predicted by Simple-II model is illustrated in Fig. 7.

Pressure drops in heater, cooler and regenerator at various crank angles are shown in Fig. 8a. As pressure drops of heat exchangers were calculated in both Simple and the Simple-II models, no difference in pressure drop prediction of the two models was observed in Fig. 8a. The results showed that the fluid friction in the regenerator had a significant contribution to pressure drop and power loss of the Stirling engine. As magnitudes of pressure drops in heater and cooler were much smaller than the corresponding pressure drop in the regenerator, on the scale of Fig. 8a, variation of these two pressure drops against crank angle were not clear; therefore, Fig. 8b is presented here to clearly show variation of

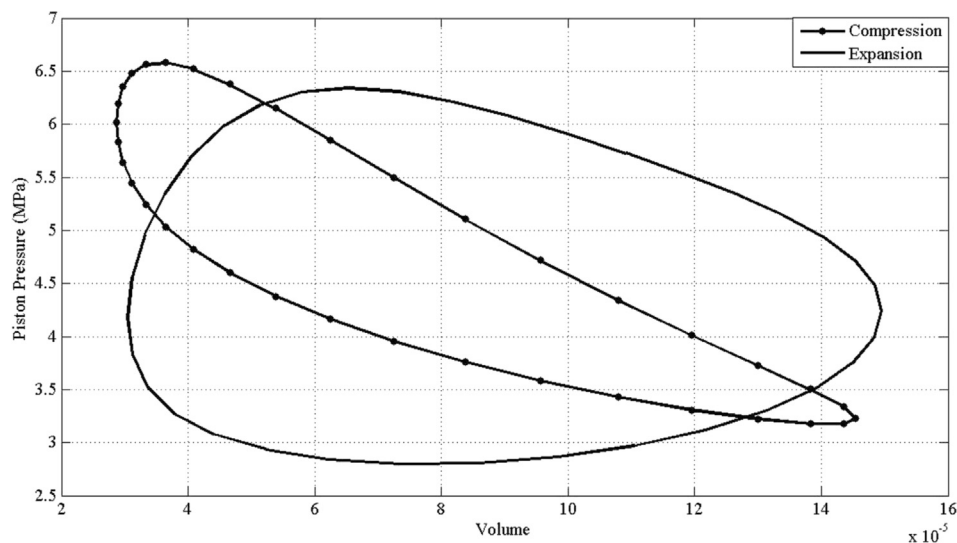


Fig. 7. Pressure distribution at various volumes of compression and expansion spaces at $f_r = 41.67$ Hz and $P_{\text{mean}} = 4.14$ MPa.

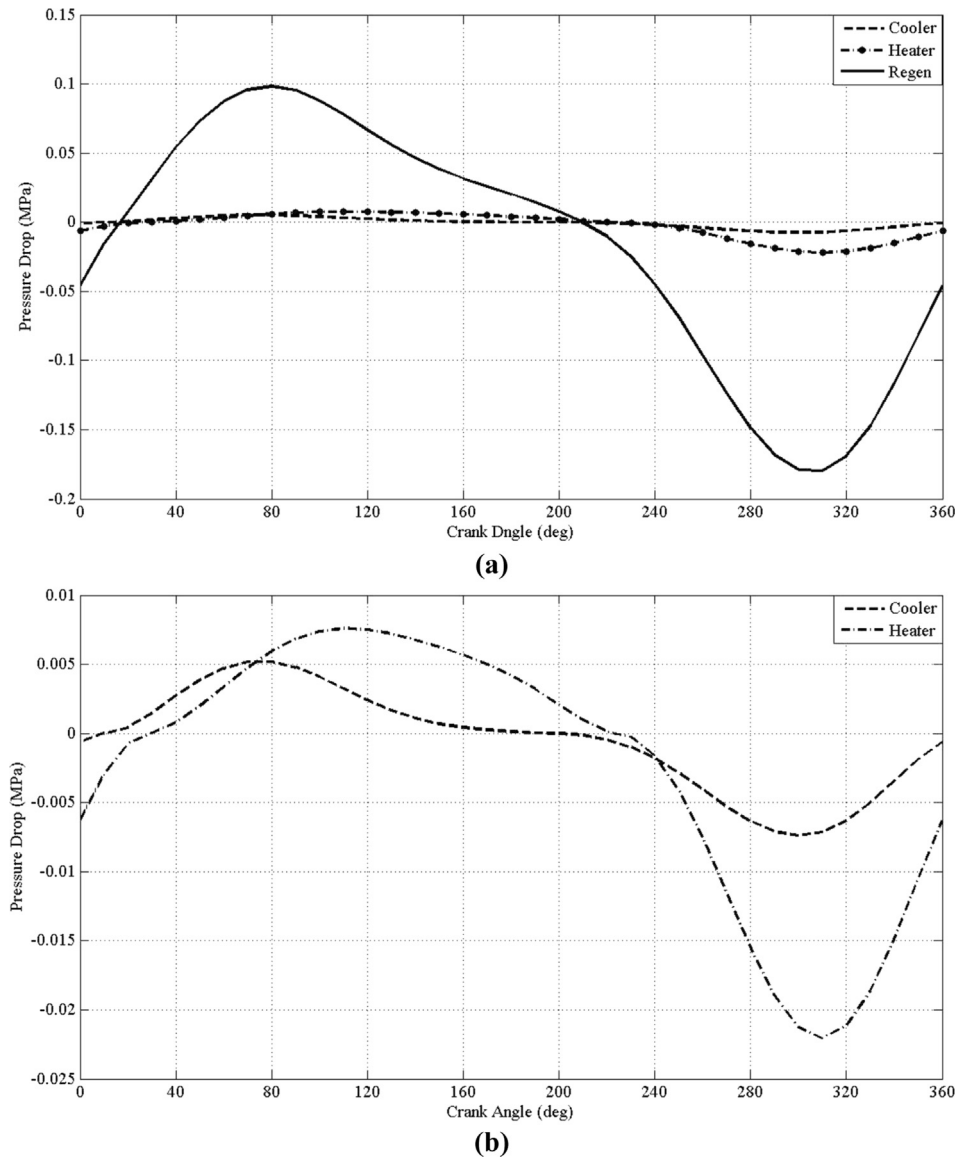


Fig. 8. (a) Pressure drops in three heat exchangers (regenerator, heater and cooler) at various crank angles at $fr = 41.67$ Hz and $P_{mean} = 4.14$ MPa; (b) pressure drops in heater and cooler at various crank angles at $fr = 41.67$ Hz and $P_{mean} = 4.14$ MPa.

pressure drops in the heater and cooler against crank angle. This figure indicates that the pressure drop in the heater was about 50% higher than the corresponding pressure drop of the cooler. Comparison of Fig. 8a and b indicated that maximum pressure drop in the regenerator was about 8.0 times of the sum of maximum pressure drops of the heater and cooler.

Pressure drop due to finite motion of pistons at various crank angles for compression and expansion spaces is depicted in Fig. 9. The results indicated that the pressure drop related to finite speed of the piston in compression space was higher than the corresponding pressure drop of the expansion space due to the fact that the working fluid temperature in the compression space was lower than the one in expansion space and finite speed pressure drop was in proportion to the inverse of the fluid temperature ($1/T$).

Comparison of Figs. 8(a, b) and 9 revealed that, although the highest pressure drop occurred in the regenerator, pressure drop due to finite speed of the piston had a reasonable value. Maximum pressure loss due to finite speed of the pistons was about 30% of the highest pressure drop of the regenerator. Moreover, as can be seen,

pressure drops due to finite speed of the piston were much higher than those of heater and cooler. For a better insight into effect of each loss mechanism on total loss of the GPU-3 Stirling engine, Table 6 is presented here.

Based on Table 6, almost 20.6% of the generated power was lost due to various power loss mechanisms, in which the most part of the power loss was due to pressure drop in heat exchangers so that 76.8% of total power loss (equal to 15.8% of generating power) was due to pressure drop in heat exchangers. The second and third portions of power loss belonged to finite speed of the piston and mechanical friction, respectively. Power loss due to gas leakage to the buffer space was 3.0% of total power loss as about 0.6% of the generated power. Moreover, as the shuttle effect was coupled to the developed system of differential equations, it was possible to evaluate its effect not only on heat loss but also for the power loss. Therefore, it was found that the shuttle effect was responsible for 0.6% of total power loss equal to 0.1% of the generated power. However, the shuttle effect had a minor role in the power loss of Stirling engine (based on Table 6); nevertheless, as is clear in

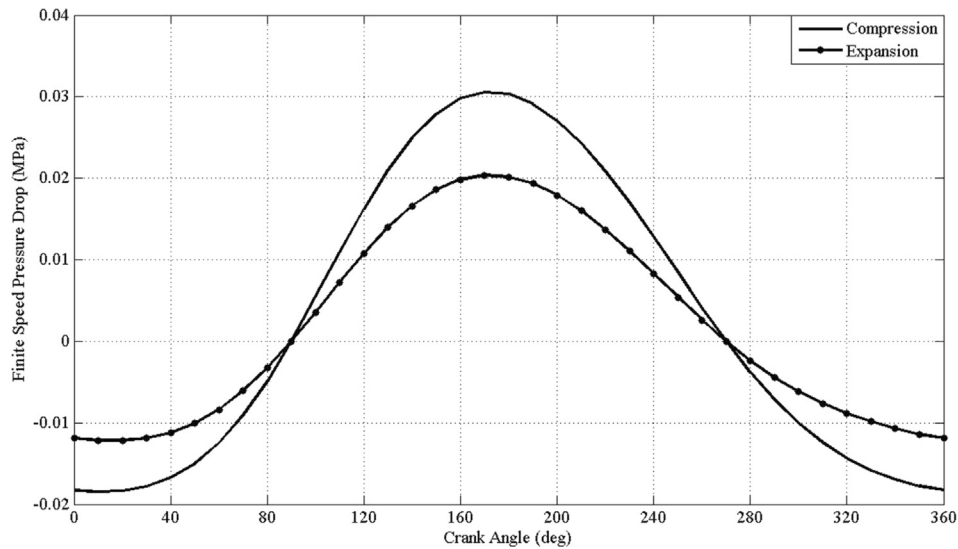


Fig. 9. Finite speed pressure drop at various crank angles for compression and expansion spaces at $fr = 41.67$ Hz and $P_{mean} = 4.14$ MPa.

Table 6

Surveying the contribution of various loss mechanisms of GPU-3 Stirling engine.

Loss mechanism		Percent of the total generated power or total heat addition	Percent of the total loss
Power losses	Power loss due to finite speed of the piston	3.2	15.2
	Power loss due to mechanical friction	0.9	4.4
	Power loss due to pressure drops in heat exchangers	15.8	76.8
	Power loss due to mass leakage	0.6	3.0
	Power loss due to shuttle effect	0.1	0.6
	Total power loss	20.6	100.0
Heat losses	Heat loss due to regenerator ineffectiveness	29.5	78.8
	Longitudinal conduction heat loss	4.5	12.0
	Heat loss due to the shuttle effect	3.0	8.0
	Heat loss due to mass leakage	0.5	1.2
	Total heat loss	37.5	100

Table 6, the shuttle loss was responsible for 8.0% of total heat loss of the GPU-3 Stirling engine (3.0% of total heat addition to engine from heat source). Moreover, it can be seen that, among the heat loss mechanisms, the greatest heat loss was due to regenerator

ineffectiveness (78.7%) followed by the longitudinal conduction loss through the regenerator wall (12.0%) and shuttle effect (8.0%). Contribution of mass leakage to total heat loss of the GPU-3 Stirling engine was 1.2%. Based on Table 6, it is clear that the regenerator

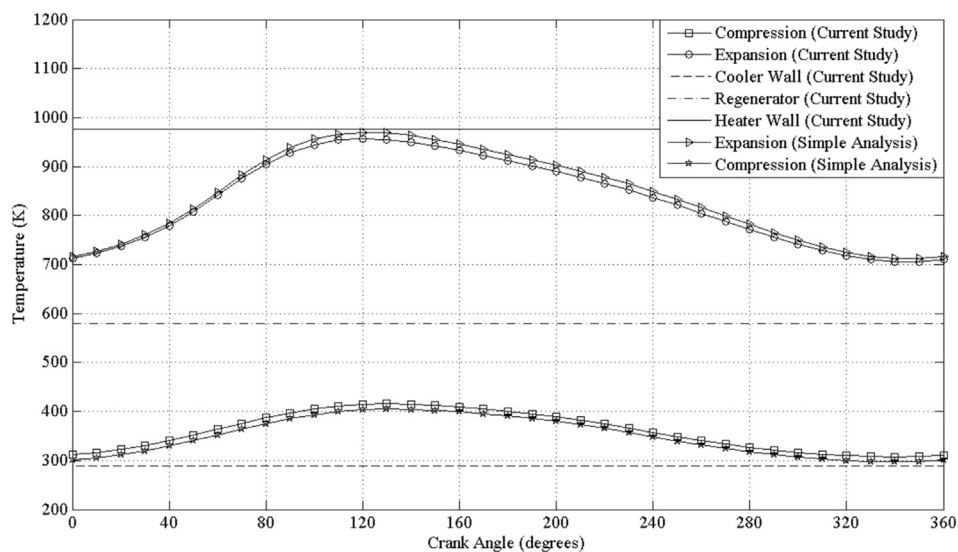


Fig. 10. Temperature distributions in heater, cooler and regenerator at various crank angles at $fr = 41.67$ Hz and $P_{mean} = 4.14$ MPa.

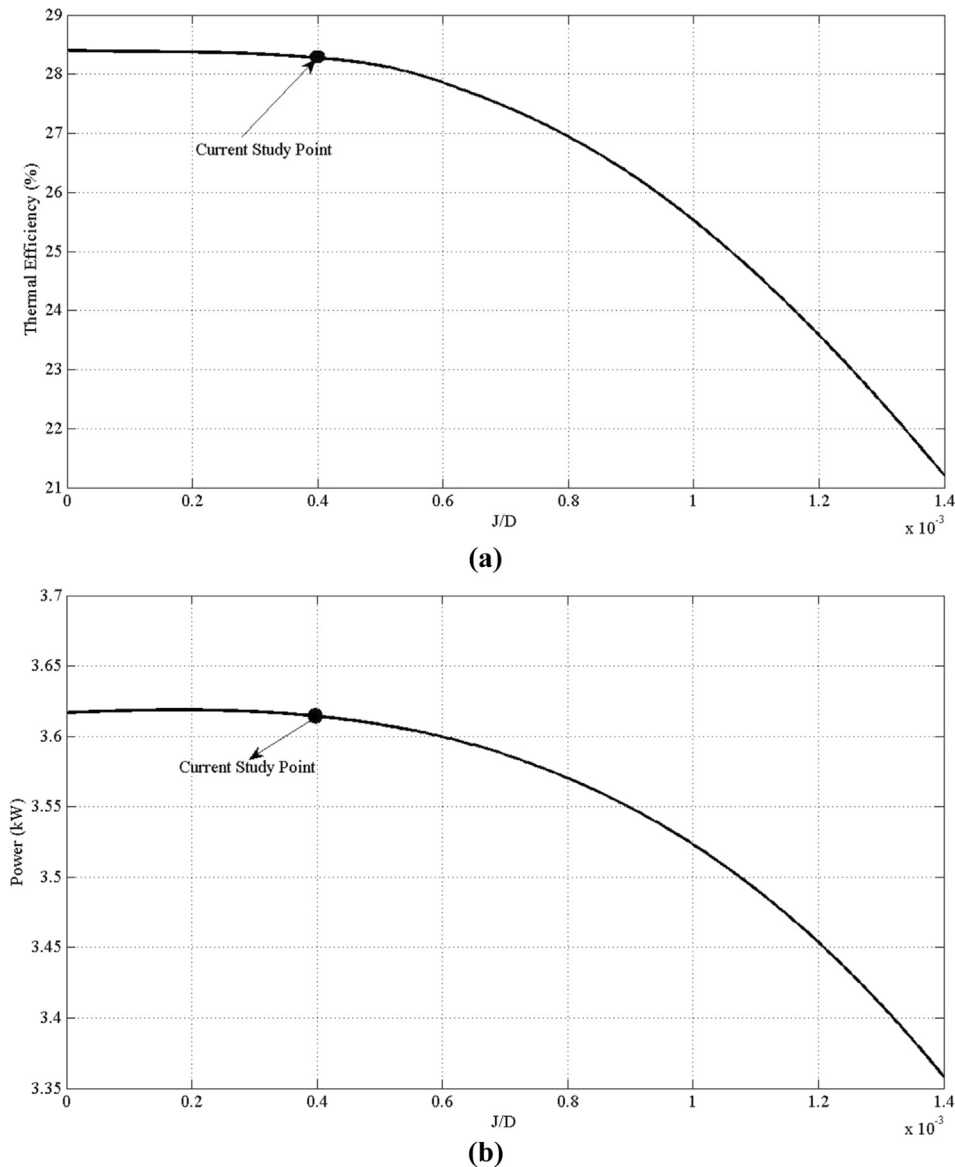


Fig. 11. Effect of mass leakage to the buffer space on (a) thermal efficiency; (b) output power ($fr = 41.67$ Hz and $P_{mean} = 4.14$ MPa).

was responsible for greater portion of power loss and heat loss. Therefore, the regenerator is a bottleneck for any thermal improvement task of Stirling engines. The role of mass leakage to buffer space was considerable in power loss, but minor in heat loss. On the other hand, shuttle effect had significant contribution to heat loss, but non-significant effect on power loss. Therefore, all the loss mechanisms considered in this paper had more or less effect on thermal performance of Stirling engines.

The temperature distribution in heater, cooler and regenerator at various crank angles is depicted based on Simple-II model and Simple analysis in Fig. 10. It can be observed that temperature of the expansion heater that was predicted by the Simple-II model was slightly less than what was predicted by the Simple analysis. On the other hand, the Simple-II model predicted slightly higher temperature in the compression space compared to the predicted compression temperature by the Simple analysis. Difference in temperature prediction by of the Simple-II thermal model and Simple analysis was due to consideration of effects of shuttle loss and longitudinal conduction loss through the regenerator wall in the Simple-II model.

Effect of cylinder-piston gap in thermal efficiency and output power of the Stirling engine is depicted in Fig. 11a and b, respectively. Results showed that thermal efficiency and power decreased with increase in the cylinder-piston gap due to increasing mass leakage. At very low values of gap dimensionless number (J/D), decreasing rate of thermal efficiency and power against increasing J/D was non-significant; but, when J/D increased to above 0.6×10^{-3} , thermal efficiency decreased drastically. At $J/D = 0.4 \times 10^{-3}$ as design J/D of the GPU-3 Stirling engine, which is shown Fig. 11a and b, effect of gas leakage to the crank case on thermal performance of the engine was non-significant.

Effect of the heater-cooler temperature difference on thermal efficiency and output power are considered in Fig. 12a and b, respectively. These figures indicate that the thermal efficiency and output power of the engine increased with increasing the heater and cooler temperature difference. Moreover, increasing rates of thermal efficiency and power were the same by both Simple-II model and Simple analysis.

As previously cited in the interpretation of Fig. 8a, the regenerator is responsible for a great part of the system power losses due

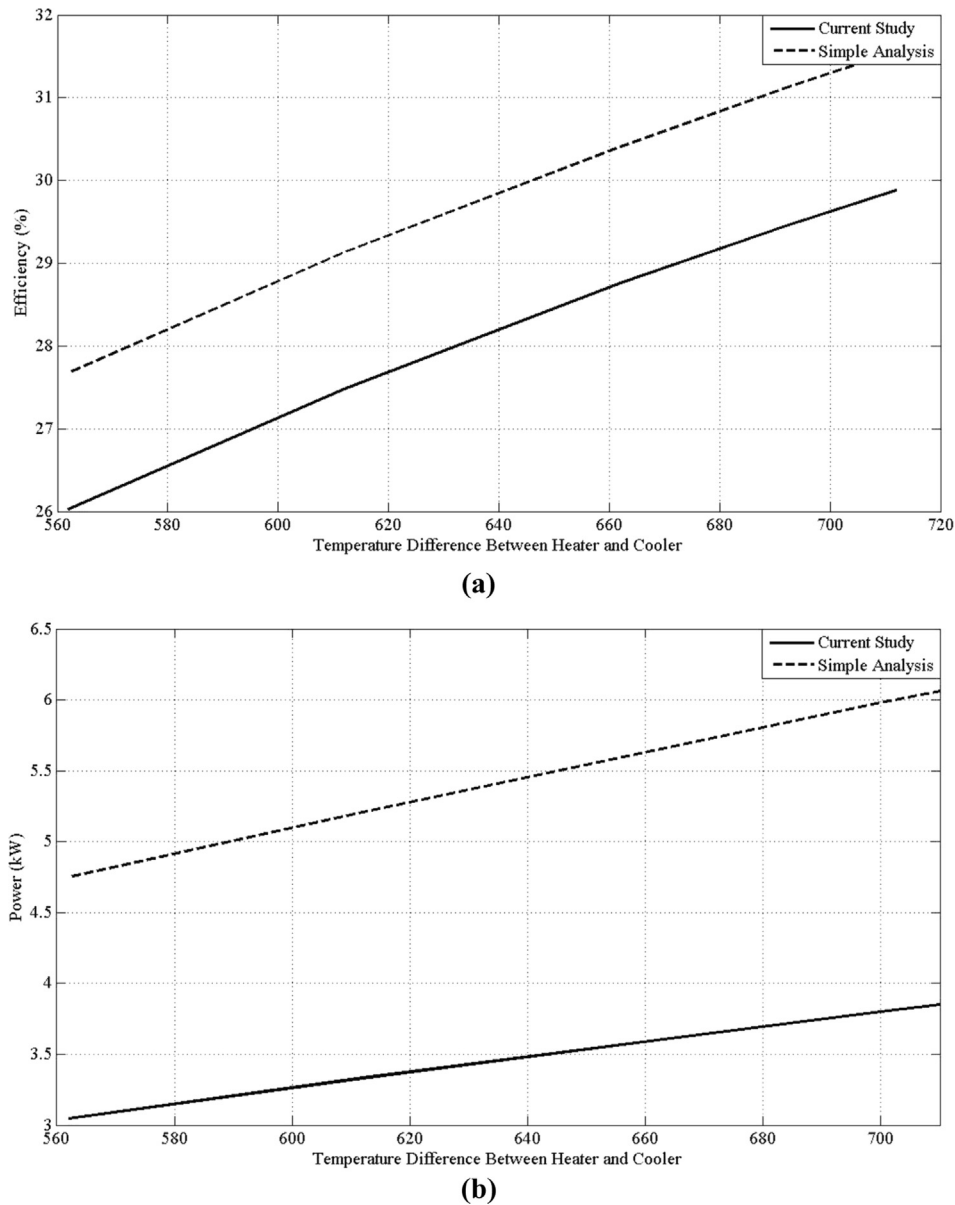


Fig. 12. Comparing effect of heater-cooler temperature difference in (a) thermal efficiency; (b) output power obtained by in Simple and Simple-II models ($\text{fr} = 41.67$ and $P_{\text{mean}} = 4.14$ MPa).

to very high-pressure drop. On the other hand, the regenerator is an essential part of Stirling engines as it recovers a great part of thermal energy of the working fluid in order to increase thermal efficiency of the engine. In addition, porosity of the regenerator mesh is a key parameter in thermohydraulic performance of the regenerator because pressure drop and heat regeneration of the regenerator are highly dependent to the porosity factor. Figs. 13a and b are presented here to show effect of porosity variation in thermal performance and output power of the GPU-3 Stirling engine, respectively. Based on Fig. 13a, increasing porosity of the regenerator (using a regenerator matrix with lower density) up to 0.80 led to increasing thermal efficiency since a reduction in porosity caused reduction in power loss. On the other hand, decreasing the regenerator porosity caused reduction of heat recovery. Therefore, as is clear in Fig. 13b, thermal efficiency–porosity curve showed a peak at $\varphi = 0.80$. Fig. 13b demonstrates that the output power increased when regenerator porosity decreased, which was due to reduction in the pressure loss in the regenerator

at lower values of porosity. Design value of GPU-3 porosity was 0.696. Therefore, if porosity of the regenerator increased from 0.696 to 0.80 (the optimum value), the predicted thermal efficiency increased from 28.4% to 32.5% and output power increased from 5.90 kW to 6.0 kW (based on output of simple-II thermal model).

Fig. 14 shows variation of the power loss against rotation frequency of the GPU-3 Stirling engine. Both Simple-II thermal model and Simple analysis predicted that power loss increased rapidly when rotation frequency of the engine increased. Based on Fig. 14, two graphs diverged when rotation frequency increased, which was due to the fact that, at higher values of engine frequency, power loss increased due to finite speed of the piston and mechanical friction. Therefore, deviation increased between power losses predicted by two models at higher frequency. At lower values of engine frequency, as power losses due to finite speed of the piston and mechanical friction became non-significant, the power losses predicted by Simple-II thermal model and Simple analysis got close to each other.

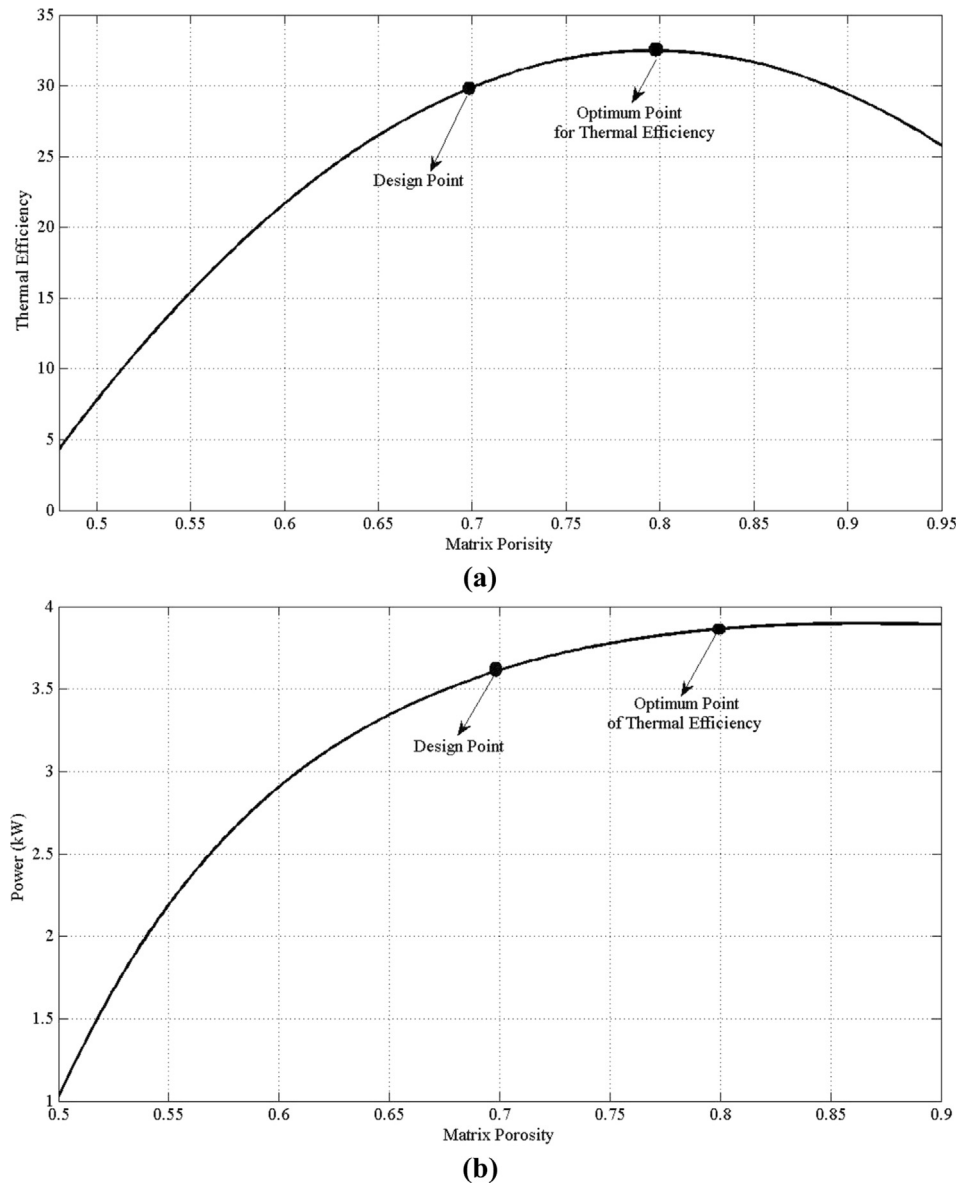


Fig. 13. Effect of regenerator matrix porosity on (a) thermal efficiency; (b) output power obtained at $fr = 41.67$ Hz and $P_{mean} = 4.14$ MPa.

6. Conclusions

The famous numerical thermal model of Stirling engines, called Simple model, was modified and a new thermal model called Simple-II, which considered various loss mechanisms including power loss due to finite speed of the piston, mechanical friction, shuttle effect, longitudinal conduction loss between heater and cooler through the regenerator wall and mass leakage, was developed. Based on the methodology presented in this work, the system of ordinary differential equation obtained in the analysis was solved using fourth-order Runge-Kutta method and results of the numerical solution were corrected for effects of various losses. Simple-II model was employed for thermal simulation of a prototype Stirling engine called GPU-3 engine. It was shown that Simple-II thermal model yielded results that were more consistent with those of the experimental data in comparison to the previous thermal models. It was also demonstrated that variation trends of thermal efficiency against variation of the engine rotation frequency and mean pressure of the Simple-II thermal model were

more consistent with the experimental trends compared to the original Simple analysis. It was observed that the prediction error of thermal efficiency was higher at higher engine rotation speeds and lower mean pressures. Based on the comprehensive analysis, it was concluded that regenerator was responsible for the greater part of power and heat losses due to pressure drop and thermal ineffectiveness, respectively. After the regenerator, the following parts or effects were responsible for the greatest values of power loss in a descending order: finite speed of piston, mechanical friction, heater pressure drop, cooler pressure drop, gas leakage and shuttle effect. Moreover, after the regenerator, the greatest part of the heat loss was due to longitudinal conduction loss through the regenerator wall followed by shuttle effect and mass leakage, respectively. It was concluded that, if the piston-cylinder gap is in a reasonable range of gaps used for designing engines, the leakage effect would have a minor effect on performance of the engine; but, at large gaps, the output power and thermal efficiency would drastically reduce. Porosity analysis of regenerator indicated that, if the current porosity of the regenerator mesh of the GPU-3 Stirling engine

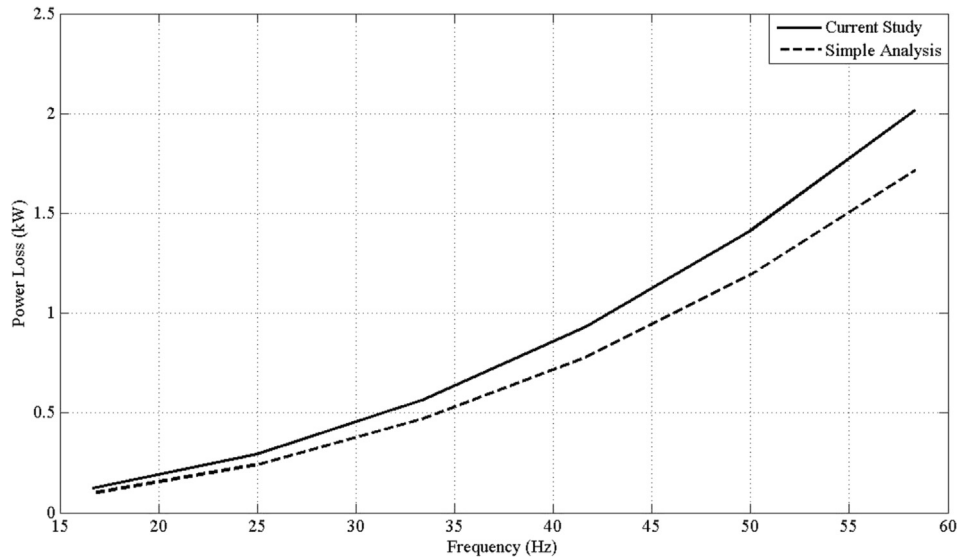


Fig. 14. Effect of rotation frequency on power loss at $P_{\text{mean}} = 2.76$ MPa.

increased by about 14.8%, the corresponding thermal efficiency and output power of the engine would increase by 4.1% (as difference) and 1.7%, respectively. As effect of power loss due to finite speed of the piston and mechanical friction was more significant at higher engine rotational speeds, the Simple-II model yielded more realistic results than the original Simple analysis at such speeds.

Nomenclature

General

A	gas cross-section area
A_{wg}	gas wetted area (m^2)
c	average speed of molecules (m s^{-1})
c_p	specific heat at constant pressure ($\text{kJ kg}^{-1} \text{K}^{-1}$)
c_v	specific heat at constant volume ($\text{kJ kg}^{-1} \text{K}^{-1}$)
d	hydraulic diameter (m)
D_d	diameter of displacer (m)
f	rotation frequency of engine (Hz)
f_{re}	Reynolds friction factor
J	annular gap between piston and cylinder (m)
K	gas conductivity
K_g	specific heat ratio [C_p/C_v]
L	length of seal (m)
L_d	length of displacer (m)
N_r	rotation speed of engine (RPM)
NTU	number of transfer unit
P	pressure (Pa)
Pr	Prandtl number
Re	Reynolds number
S	displacer stroke (m)
St	Stanton number
u	velocity of gas flow (m s^{-1})
u_p	Piston linear velocity (m s^{-1})
V	volume (m^3)
R_{cond}	conduction resistance (m K W^{-1})

Greek

ρ	density (kg m^{-3})
ε	regenerator effectiveness
μ	viscosity (Pa s)

Subscript

c	cooler
ck	cooler–compression space interface
e	expansion space
h	heater
he	heater–expansion space interface
hi	hot gas input
ho	hot gas output
k	Kooler (cooler)
kr	Kooler (cooler)–regenerator interface
leak	leakage
r	regenerator
rh	regenerator–heater interface
shuttle	shuttle effect
wh	heater wall
wk	cooler wall

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